February 2025
Issue 1

CSNP: Economic Decision-Making Under Uncertainty

University Team Note

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Introduction

We require a **common framework** within which different possible approaches may be compared.

The present project is concerned with possible replacements for *least worst regret* (LWR) analysis, which is a *single-point-in-time* decision-making technique. Nevertheless, in applying LWR or any other technique, account needs to be taken of possible future decision-making opportunities, both in evaluating possible future paths and in determining their associated costs.

Assessment

The formal framework associated with decision-making at a single point in time is as follows. We have:

- An **uncertainty space** S consisting of various possible evolutions of the future—here referred to as scenarios—together with some specification of their relative likelihoods;
- A **set** D of **possible decisions** (continuous or discrete);
- A **cost** c_{ij} (either a monetary cost, or some more generalised cost or negative utility) associated with each point $j \in S$ in the uncertainty space and each possible decision $i \in D$.

There is thus, formally, a **matrix of costs** c_{ij}, indexed in one dimension by **possible uncertainties** (with some measure of their relative likelihoods) and in the other dimension by **possible decisions**.

It is also convenient, for each scenario $j \in S$, to define the corresponding regret function

$$r_{ij} = c_{ij} - \min_{i' \in D} c_{i'j}\,, \qquad (1)$$

over the set D of possible decisions i. Within this framework, there are various possible approaches.

(Robust) Bayesian Analysis

In a Bayesian analysis, a probability p_j is assigned to each scenario $j \in S$, and the optimal decision i is simply that decision which minimises the corresponding expected cost

$$\sum_{j \in S} p_{ij} c_{ij}. \qquad (2)$$

Usually, the costs c_{ij} are monetary or economic, but they might be more generalised as described above. For the purpose of comparison with LW(W)R analysis below, we note that it follows from (1) and (2) that the Bayesian optimal decision i is also that which minimises the corresponded **expected regret**

$$\sum_{j \in S} p_{ij} r_{ij}, \tag{3}$$

i.e., the expected cost and expected regret functions differ by a constant as the possible decisions i are varied.

In principle, a Bayesian analysis is the only scientifically rational approach. Savage's Theorem [1] essentially shows that, under reasonable axioms of rational behaviour, a preferred decision will correspond to a Bayesian analysis with some assignment of probabilities and *some* suitably defined cost or utility function.

However, in practice, both possible evolutions of the future and their associated probabilities are often very uncertain, and a model cannot in any case cover everything that might happen—*the more so since some events may be entirely unforeseen*. Further, different interested parties may have different opinions about possible futures and their probabilities, and also about future needs and costs. It is therefore important to make the analysis **robust** by conducting appropriate **sensitivity analyses** with respect to all features of the modelling – the *scenario set* (set of possible evolutions of the future), the associated *probabilities* and the *costs*. The aim should be to obtain a decision which is *robust* under variation of assumptions. By *robust*, we mean that the preferred decision is either optimal, or remains close to optimal, as these assumptions are varied.

It is also important to note that the Bayesian approach naturally extends to allow for future decision-making opportunities: the evolution of the future has a branching structure and, in determining costs associated with immediate decisions, account may naturally be taken of this.

Least Worst (Weighted) Regret

In least worst (weighted) regret analysis the optimal decision i is simply that decision which minimises the corresponding maximum (weighted) regret

$$\max_{j \in S} w_j r_{ij}\ , \tag{4}$$

Within standard **LWR** analysis, the scenario weights w_j are all taken to be equal and no attempt is made to specify the relative likelihoods of the scenarios. It is for this reason that the technique is popular, as it is considered to be *objective*. However, this objectivity is illusory (see [2]), as the optimal decision depends crucially on the choice of scenario set, and each scenario one might consider can only be taken to be either inside or outside the scenario set S. At a minimum, it is essential that the scenarios are sufficiently representative of the set of all possible futures, and that each scenario is individually reasonably plausible.

LWWR analysis [3] attempts to get round this difficulty by allowing *weights* w_i to be assigned to scenarios. While these *weights* may be normalised to sum to 1, we are unable to find any interpretation of them as *probabilities* in the usual sense of that word, i.e. there is no probabilistic theory which would then lead to the *LWWR* criterion, in which the *expectation* operation in (3) is replaced by the weighted *maximisation* operation in (4).

For this reason, there is no choice of weights within *LWWR* analysis that necessarily produces the same answer as a *Bayesian* analysis. We are further unable to find any presentation of a scientific basis for *LWWR*, either in [3] or elsewhere, and the problem of assigning weights necessarily requires subjective judgements, as in Bayesian approaches.

The difficulty is highlighted by the observation that *LWR* analysis, and more generally *LWWR* analysis, fails to satisfy the economically and logically desirable independence of irrelevant alternatives property (which is naturally satisfied within any *Bayesian* analysis). A consequence of this failure is that which of two decisions is preferred may depend on the data associated with a third possible decision, even though the latter is not itself sensible. Indeed, within *LWR* or *LWWR* it is generally possible to make any decision the preferred decision by introducing a further decision into the set S with carefully chosen costs–see [2], Proposition 3. For this reason, *LW(W)R* analysis is open to **gaming** by interested parties in order to achieve desired results.

Finally, it is difficult to see how *LWWR* analysis, again unlike *Bayesian* analysis, may be extended to repeated decision-making over extended periods of time.

An example of the general unsatisfactoriness of LWR analysis, and by extension LWWR analysis, arises within the determination of a *capacity-to-procure* for the *electricity capacity market*. LWR analysis simply corresponds to a weighted average of the optimal decisions corresponding to the most optimistic and the most pessimistic of the scenarios or sensitivities chosen for analysis–choices which might well be made on the basis of obtaining a seemingly reasonable answer. With *LWWR* analysis, nothing changes until the most extreme scenarios/sensitivities are sufficiently downweighted, at which point they simply drop out of the analysis entirely, so that the answer is then as given (unweighted) *LWR*, but with a different choice of scenario set.

Stochastic Optimisation

The term *stochastic optimisation* has many different meanings, but, insofar as it is relevant to decision-making, it generally refers to optimisation within models that have (usually well-specified) probabilistic components. Insofar as this in relevant to the decision-making being considered within the CSNP—where the uncertainties arise mainly with respect to various ways in which the future might evolve, stochastic optimisation is essentially another term for Bayesian analysis, which is dealt with above.

Deep Uncertainty

Other decision-making approaches are largely concerned with the management of deep uncertainty [4, 5] where the uncertainty space may be very complex, with, for example, many branching points in the future and many possibilities for the occurrence of unforeseen events, and where it may be difficult to specify probabilities, particularly with regard to those events which might happen in the more distant future. In particular, it may be impossible to obtain agreement among decision-makers on these matters.

Here, while the decision-making philosophy remains in *principle* a robust Bayesian one, it may be necessary to navigate very differently the exploration of the interaction between the uncertainty and decision spaces. Thus, for example, it may be sufficient to bound many of the probabilities involved without the need to specify them precisely, and some deep uncertainty approaches do not attempt to formally specify probabilities at all—although of course relative likelihoods continue to be important. It is further the case that such deep uncertainty approaches usually rely on a continuity of interaction with the decision-makers over extended periods of time.

With regard to the present applications (infrastructure decision-making for NESO), it seems to us that both the uncertainty space and the decision space are too well-structured to require this approach.

Summary

It seems to us that, in the present project, possible evolutions of the future are sufficiently well understood, and possible decisions sufficiently limited and clearly defined, that an explicitly probabilistic (robust Bayesian) approach is to be preferred. In particular, this permits at least some quantification of future uncertainties - for example, with regard to technological progress, the economy, the speed of climate change, or political developments - and the ability to incorporate such information is clearly important. However, it seems essential to use an explicitly robust approach, e.g., via the use of *Bayesian sensitivity analysis*.

References

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